Chapter 10 - Solved Problems

Solved Problem 10.1. *Assume that the disturbance in a control loop has the form*

$$d_g(t) = A\left(\cos(2t)\right)^2 \tag{1}$$

Determine the generating polynomial.

Solutions to Solved Problem 10.1

Solved Problem 10.2. *Consider a plant having a nominal model given by*

$$G_o(s) = \frac{1}{s+1} \tag{2}$$

Assume that this plant must be controlled in closed loop to follow a constant reference. It is also known that the output disturbance, $d_g(t)$, can be characterized by

$$d_g(t) = K_1 + K_2\cos(2t+\alpha) + v(t) \tag{3}$$

where $v(t)$ is a varying component with significant energy only in the frequency band $[0; 3]$ $[rad/s]$. *Synthesize a controller which provides zero steady steady error and fast disturbance compensation.*

Solutions to Solved Problem 10.2

Solved Problem 10.3. *Consider the control architecture depicted in Figure 1 which implements an alternative reference feedforward scheme.*

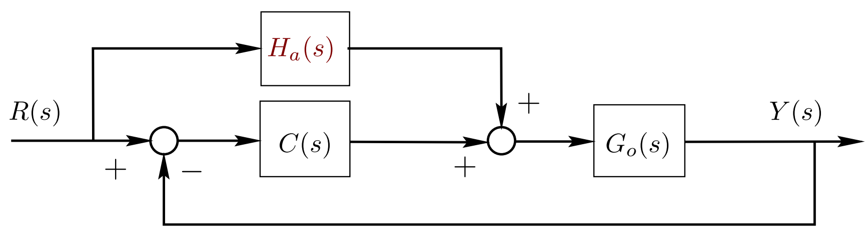

Figure 1: Alternative reference feedforward architecture

10.3.1 *Find $H_a(s)$ in Figure 1 such that the tracking properties are the same as those in the control loop shown in Figure 10.10 of the book.*

10.3.2 *Show that the ideal choice for $H_a(s)$ is $G_o(s)^{-1}$.*

Solutions to Solved Problem 10.3

Solved Problem 10.4. *Consider the disturbance feedforward architecture in Figure 10.2 of the book. Assume that $d_g(t) = A\cos(\omega_o t)$ and that*

$$G_{o1}(s) = \frac{e^{-0.2s}}{s+0.8} \tag{4}$$

Further assume that $G_f(s) = -0.8$. *Will this design yield a better performance regarding disturbance compensation than that of a simple one degree of freedom control loop?*

Solutions to Solved Problem 10.4

Solved Problem 10.5. *Consider the cascade control architecture shown in Figure 10.8 of the book. Assume that*

$$G_{o1}(s) = 1; \quad G_a(s) = \frac{1}{s+1}; \quad G_b(s) = \frac{e^{-2s}}{3s+1} \tag{5}$$

Further assume that the reference, $r(t)$, *and the disturbance,* $d_g(t)$, *are step-like signals.*

10.5.1 *Design* $C_1(s)$ *and* $C_2(s)$ *to achieve good disturbance rejection and a complementary sensitivity given by*

$$T_o(s) = \frac{9e^{-2s}}{s^2+5s+9} \tag{6}$$

10.5.2 *Compare the performance with that of a one degree of freedom controller which achieves the same complementary sensitivity* (6).

Solutions to Solved Problem 10.5

Chapter 10 - Solutions to Solved Problems

Solution 10.1. *Using trigonometric identities the disturbance can be expressed as*

$$d_g(t) = 0.5A + 0.5A\cos(4t) \tag{7}$$

This leads to

$$D_g(s) = \mathcal{L}\left[d_g(t)\right] = \frac{0.5A}{s} + \frac{0.5As}{s^2+16} \tag{8}$$

Therefore, the generating polynomial is given by

$$\boxed{\Gamma_d(s) = s(s^2+16)} \tag{9}$$

Solution 10.2. *We will use pole assignment techniques with the following specifications*

a) *The controller must have integration to deal properly with the constant reference and the constant disturbance component.*

b) *The controller must have poles at $s = \pm j2$ to force complete compensation (in steady state) of the sinusoidal disturbance component.*

c) *The minimal degree of the closed-loop polynomial $A_{cl}(s)$ is therefore equal to $2n - 1 + 1 + 2 = 4$*

d) *The closed-loop bandwidth should be larger than the frequency band of the varying component of the disturbance $v(t)$.*

With the above specifications we choose

$$A_{cl}(s) = (s^2+7s+25)(s+8)(s+10) = s^4 + 25s^3 + 231s^2 + 1010s + 2000 \tag{10}$$

We can now solve the Diophantine equation

$$\underbrace{(s+1)}_{A_o(s)}\underbrace{s(s^2+4)}_{L(s)} + \underbrace{1}_{B_o(s)}\underbrace{(p_3s^3+p_2s^2+p_1s+p_0)}_{P(s)} = s^4 + 25s^3 + 231s^2 + 1010s + 2000 \tag{11}$$

The solution of this equation can done with the MATLAB function ***paq.m*** *distributed with the book and available on the web site.*

```
>>Ao=[1 1];Am=conv(Ao,[1 0 4 0]);Bo=1;Acl=[1 25 231 1010 2000];
>>[Lm,P]=paq(Am,Bo,Acl); L=conv(Lm',[1 0 4 0]);C=tf(P',L);
```

The solution to (11) is

$$P(s) = 24s^3 + 227s^2 + 1006s + 2000 \tag{12}$$

$$C(s) = \frac{24s^3 + 227s^2 + 1006s + 2000}{s(s^2+4)} \tag{13}$$

Solution 10.3. *We first notice that feeding the reference through $H_a(s)$ is equivalent to having an input disturbance equal to $H_a(s)R(s)$. Hence the output can be expressed as*

$$Y(s) = T_o(s)R(s) + S_{io}(s)H_a(s)R(s) \tag{14}$$

10.3.1 *Equation (14) can also be written as*

$$Y(s) = T_o(s)\left(1 + \frac{S_{io}(s)}{T_o(s)}H_a(s)\right)R(s) \tag{15}$$

Hence, if we compare this result with the tracking characteristic in Fig 10.10 of the book, $Y(s) = T_o(s)H(s)R(s)$ we have that

$$H(s) = 1 + \frac{S_{io}(s)}{T_o(s)}H_a(s) = 1 + \frac{H_a(s)}{C(s)} \tag{16}$$

10.3.2 *From (14) and the fact that $T_o(s) = 1 - S_o(s)$ we have that*

$$Y(s) = (T_o(s) + S_o(s)G_o(s)H_a(s))\,R(s) = R(s) + (G_o(s)H_a(s) - 1)S_o(s)R(s) \tag{17}$$

Thus, the ideal choice for $H_a(s)$ is the inverse of the plant nominal transfer function.

Solution 10.4. *With disturbance feedforward, the output (due to the disturbance) is given by*

$$Y(s) = S_{io}(s)(1 + G_f(s)G_{o1}(s))D_g(s) \tag{18}$$

On the other hand, if no disturbance feedforward is used then

$$Y(s) = S_{io}(s)D_g(s) \tag{19}$$

Thus, the disturbance feedforward will yield better performance if and only if

$$|1 + G_f(j\omega_o)G_{o1}(j\omega_o)| \leq 1 \tag{20}$$

In this problem

$$1 + G_f(s)G_{o1}(s) = 1 - \frac{0.8e^{-0.2s}}{s + 0.8} \tag{21}$$

The frequency response of this factor is shown in Figure 2

From Figure 2 we observe that this feedforward design will improve the disturbance compensation for $\omega_o < 1.3$. However, it is counterproductive if $1.3 < \omega_o < 15$. For $\omega_o > 15$, the performance is approximately the same as the case where no disturbance feedforward is used.

Solution 10.5. *The problem will be tackled using a Smith controller to build the primary controller, for both architectures (see section §7.4 of the book).*

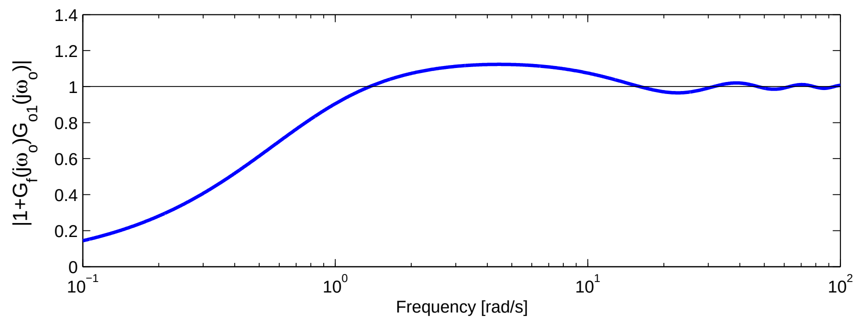

Figure 2: Frequency response

10.5.1 *For the cascade architecture, we choose to design $C_2(s)$ so as to achieve disturbance compensation with the same dynamics as that prescribed in (6). To that end, we choose $C_2(s)$ as a PI controller given by*

$$C_2(s) = \frac{4s+9}{s} \tag{22}$$

Then the secondary-loop complementary sensitivity, $T_{o2}(s)$, is

$$T_{o2}(s) = \frac{4s+9}{s^2+5s+9} \tag{23}$$

and the equivalent plant is then

$$G_{oeq}(s) = G_b(s)T_{o2}(s) = \underbrace{\frac{4s+9}{(3s+1)(s^2+4s+9)}}_{\overline{G}_{oeq}(s)} \; e^{-2s} \tag{24}$$

Then the controller $C_1(s)$ is implemented in Smith controller form to achieve (6). The resulting structure is shown in Figure 3.

The prescribed complementary sensitivity (6) is achieved if $C_b(s)$ is

$$C_b(s) = \frac{9(3s+1)(s^2+5s+9)}{s(4s+9)(s+5)} \tag{25}$$

The final cascade design is evaluated with a unit step reference and a unit step disturbance. The results are shown in Figure 4. It can be seen that excellent disturbance rejection is achieved.

10.5.2 *A standard one d.o.f. control loop is next designed. This is also built using a Smith controller, with the structure shown in Figure 5. To achieve (6) the controller $C_c(s)$ has to be chosen equal to*

$$C_c(s) = \frac{9(3s+1)(s+1)}{(s(s+5)} \tag{26}$$

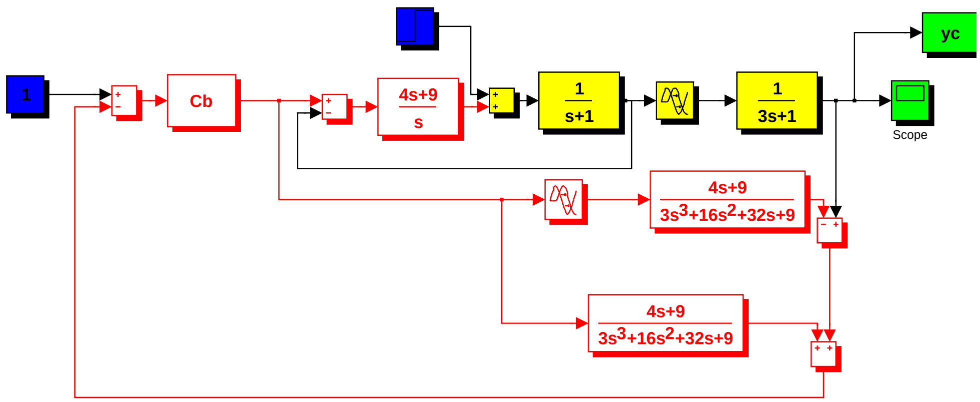

Figure 3: Cascade control with Smith controller

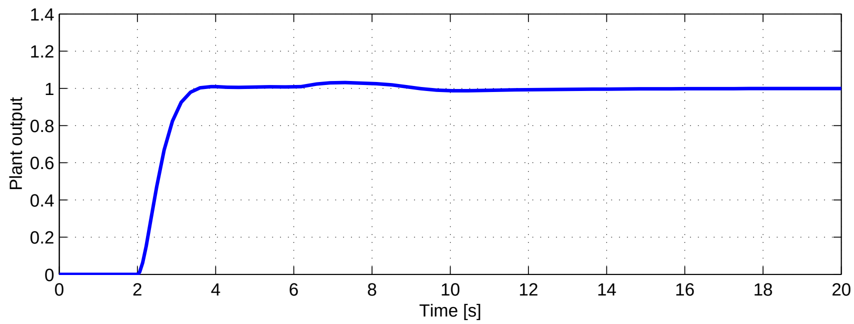

Figure 4: Cascade closed-loop response with $r(t) = \mu(t)$ and $d_g(t) = \mu(t-4)$

The performance of this design is then assessed by simulating the loop response to the same reference and disturbance as those used in the cascade design evaluation.

The results are shown in Figure 6. It can be seen that relatively poor disturbance rejection is achieved. This performance is certainly inferior to that obtained with the cascade control design.

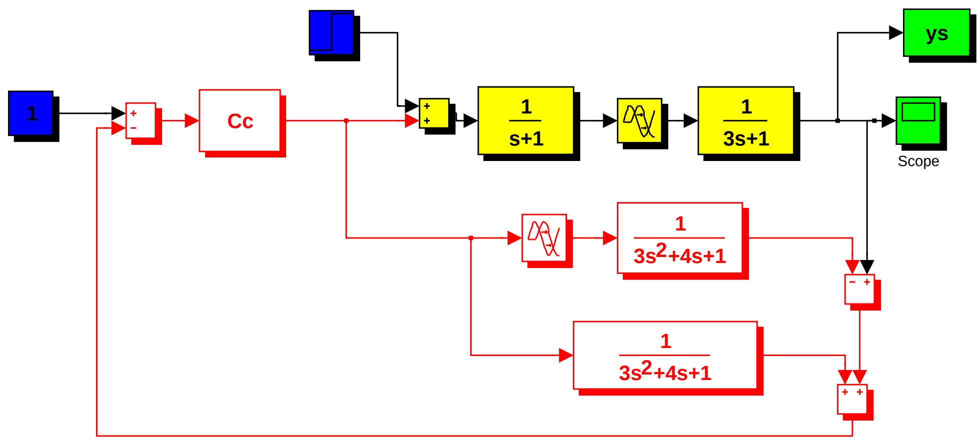

Figure 5: One degree of freedom with Smith controller control

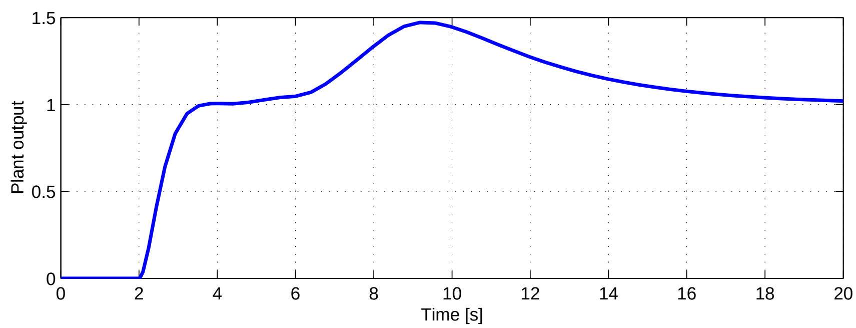

Figure 6: Cascade closed-loop response with $r(t) = \mu(t)$ and $d_g(t) = \mu(t-4)$