Chapter 20 - Solved Problems

Solved Problem 20.1. *Contributed by - Nazli Gundes, University of California, Davis, USA.*
Let

$$G_o = \begin{bmatrix} \dfrac{s-1}{(s+1)^2} & 0 \\ \dfrac{2}{s+1} & \dfrac{1}{s-1} \end{bmatrix}. \tag{1}$$

Define $Q := C(I + G_oC)^{-1}$*, i.e.,* $C = Q(I - G_oQ)^{-1}$*, where*

$$Q = \begin{bmatrix} 0 & 1 \\ \dfrac{s-1}{s+1} & 0 \end{bmatrix}. \tag{2}$$

Show that whilst S_o*,* T_o*,* S_{uo} *and* S_{io} *are all stable, that* $S_{uo}G_o$ *is not.*

Solutions to Solved Problem 20.1

Chapter 20 - Solutions to Solved Problems

Solution 20.1. *We have*

$$T_o = G_oQ = \begin{bmatrix} 0 & \frac{s-1}{(s+1)^2} \\ \frac{1}{s+1} & \frac{2}{s+1} \end{bmatrix}, \quad S_{uo} = Q, \tag{3}$$

$$S_{io} = (I - G_oQ)G_o = S_oG_o = \begin{bmatrix} \frac{(s-1)^2}{(s+1)^3} & -\frac{1}{(s+1)^2} \\ \frac{(s-1)(2s+1)}{(s+1)^3} & \frac{1}{s+1} \end{bmatrix} \tag{4}$$

are all stable. But $-S_{uo}G_o = -QG_o$ is not stable, as can be seen from

$$-S_{uo}G_o = -\begin{bmatrix} \frac{2}{s+1} & \frac{1}{s-1} \\ \frac{(s-1)^2}{(s+1)^3} & 0 \end{bmatrix} \tag{5}$$

where $-S_{uo}G_o$ is the closed-loop transfer-function from the input D_i to the output U in fig. 20.3, page 624 of the book.

(Note the Errata given for Lemma 20.2)
